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REVIEWS

One-Dimensional Two-Phase Flow. By G. B. Wallis. McGraw Hill, 1969. 408 pp. £7. 18*s*.

Cocurrent Gas-Liquid Flow. Edited by E. Rhodes and D. S. Scott. Plenum Press, 1969. 698 pp. $27.50.

Two-phase flow has been the subject of active research interest for the last 25 years and during this time several thousand papers must have been written. Besides original work, many excellent review articles have appeared, but until now no one has produced an elementary, general text-book on two-phase flow suitable for recommendation to graduate students. Professor Wallis has now rectified this omission.

The book is divided into two sections entitled 'Analytical techniques' and 'Practical applications'. In the first section Professor Wallis considers the underlying principles behind two-phase flow and develops in detail specific models such as the homogeneous model and the drift flux model. Wherever possible the development is general, reference to the various régimes of flow being kept to a minimum. This section is the main achievement of the book. Few people tried to develop an overall picture of the subject, and it is a most refreshing change for someone to stand back and view the subject as a whole, instead of concentrating on a minute detail.

The second section deals with each flow régime separately; suspended particles, bubbles, slugs, annular flow and drop flow. In style these chapters are similar to the familiar review articles. Experimental results and well-known correlations are presented as in the comparison with the general results of § 1. Sometimes, however, alternative correlations are presented with confusing rapidity and the reader is at a loss to distinguish between the accurate and not so accurate. More comment at this stage, even if it only reflected the author's prejudices, would have been welcome. None the less, this section is a most useful compilation of experimental results, especially as it contains much of the author's own work. Professor Wallis's habit of presenting his work at conferences or as internal industrial reports has resulted in much of his work being available only in rare or expensive publications. It is very nice to have a summary of his contributions to the subject in convenient form.

Professor Wallis has attempted to produce a unified nomenclature for the whole of two-phase flow. One hopes that this laudable attempt will prove popular, though the use of some unfamiliar symbols, such as j for superficial velocity, and the possibility of a letter having four subscripts and two superscripts as well as being raised to some power, will no doubt prove a deterrent. This objective will not be helped by the printing of this book. The publishers seem to consider that size, not position, is the essential for a subscript and it is often difficult to decide whether a letter is a subscript to the previous letter or not. Apart from the algebra the book is well produced and clearly printed.

It is perhaps inevitable that an author resident in America should use the former 'British' system of units, but the author of an undergraduate text-book should surely set an example by using consistent units. The following units for mass velocity were noticed: lb./(h) (ft.^{2}), lb./(ft.^{2})(sec), slugs/(sec)(ft.^{2}), lbm./(ft.^{2}) (sec), lbm./(sec)(in.^{2}), lb./sec-ft.^{2} and g/(cm^2)(sec). cfm, fps, psi and even psi^{-1} are used, randomly interspersed with the same quantities in more basic form. The book will convince any Briton who is hesitant about the current change to the metric system.

My main criticism of the book, however, is directed against the selection of material. It is inevitable, and perhaps even desirable, that emphasis should have been placed on Professor Wallis's own contributions to the subject, but the total omission of any reference to heat transfer in such a book is most surprising. On the other hand, waves, both dynamic and continuity, are dealt with at length. Since in the former case the results are often sensitive to the unknown rate of equilibration of the phases, the utility of the analyses is questionable. The presentation of this section is first-rate, but even so the book would have been improved by the replacement of part of the section on waves by an account of heat transfer.

This book gives a convenient and detailed summary of some important aspects of two-phase flow and despite the units and incomplete coverage will be welcomed by those wishing to obtain an introduction to the subject. The price of £7. 18*s*. will, however, detract from its popularity as a graduate text.

Cocurrent Gas-Liquid Flow is a collection of 27 papers originally presented at a symposium at the University of Waterloo. This was timed to precede the Joint Meeting of the American, Canadian and British Institution of Chemical Engineers and the organizers were thereby able to attract a truly worldwide body of contributors.

A popular field of study was the flow (either adiabatic or with heat transfer) in unusual geometries. There are papers dealing with the flow in grooved tubes, bends, eccentric annuli, capillary tubes, helical coils and multi-rod bundles. The experiments were carried out with meticulous care but the results seem to be specific to the particular geometry and, though no doubt valuable to the sponsoring authority, have little general interest. An exception is the work on eccentric annuli which may well be capable of generalization, especially the results for the circumferential transport of liquid.

Of more general interest is a group of papers that consider the interpretation of model tests. The effects of scale-up in bubble flow, the comparison of the pressure gradients in air-water and steam-water systems, the differences in heat-transfer coefficient between upflow and downflow and the use of water to model boiling of alkali metals are all considered. In general, however, the interpretation of model tests does not seem to be fully understood.

Most of the authors who presented experimental work are clearly first-rate experimentalists but many of them are weak on presentation and interpretation. The well-known device of plotting the square root of the experimental reading on logarithmic co-ordinates is still popular and the reasons for the choice of correlating parameters (such as the *sum* of the mass flow rate and the temperature difference) are not always made clear. Too often the quantity of interest is combined with so many other variables that the significance of the graphical correlations is quite lost.

On the theoretical side, considerable success was achieved in the prediction of the propagation velocity of pressure waves in gas-liquid mixtures and in the gas velocity for the inception of surface waves on an annular film. In the latter case, however, the results are valid only for infinitesimal waves. The equations for a particular model of mass transfer with back mixing have been solved and the results tabulated. Workers on systems to which this model is applicable will find the tabulations a great convenience.

A most interesting suggestion is made by Professor Nicklin. He shows that, for upward cocurrent annular flow to be stable, the film must be in some critical condition and he suggests that this might have analogies with flooding and countercurrent flow. It is too soon to tell whether this suggestion will be fruitful but it seems to be a line worth pursuing.

The main result of this conference has been the collection of much detailed experimental information but there are disappointingly few new ideas. This is the sort of book to which many people would like access from time to time but few would feel the need for a personal copy.

R. M. NEDDERMAN

Non-equilibrium Flows, Part I. Edited by P. P. WEGENER. Marcel Dekker, 1969. 255 pp. $13.75 or £6 11*s*.

This is the first publication in a projected, apparently open-ended, series on Gasdynamics. The series is stated in the preface to differ from both the Progress and Advances series as well as from proceedings of symposia or review articles. It is not really a collection of advanced textbooks either, although the individual articles have much in common with short advanced texts.

The subjects covered in Part I are 'Small-disturbance theories,' by Clarke; 'One-dimensional flow and compression waves,' by Becker and Böhme; 'Relaxation in gas-particles flow', by Rudinger; and 'Expansions with condensation and homogeneous nucleation of water vapour,' by Wegener.

All four articles treat a fairly narrow subject in considerably more detail than available text books, and all articles, in particular the last two, contain much new material not hitherto generally available.

All the articles are well written and well presented and the book can be warmly recommended to both experts and new-comers. It is unfortunate that very few individuals will be prepared to pay such a high price for a slender volume.

N. H. JOHANNESEN

Numerical Methods for Partial Differential Equations. By W. F. Ames. Nelson, 1969. 291 pp. £3. 5*s*.

Computational Methods in Partial Differential Equations. By A. R. Mitchell. Wiley, 1969. 255 pp. £4 (cloth) or £2. 5*s*. (paper).

Around 1950 von Neumann expressed a belief that the non-linear problems of fluid mechanics could be solved on computers. His sanguine hopes have not yet been properly realised, partly because machines are only now becoming powerful enough to cope with three-dimensional problems. Yet 'we are more often limited by inadequacies of the mathematical (including numerical) methods than by inadequacies of the computers', as Richtmyer and Morton pointed out. Their classic study of difference methods for initial-value problems has been followed by a small stream of books on the numerical solution of partial differential equations; and here are two books appearing at the same time, primarily aimed at advanced undergraduate students in engineering, physics and mathematics.

Any such student should have access to a digital computer and should be introduced to numerical analysis. Thus he should learn about finite difference methods and the perils associated with them: no-one who has used an unstable method and seen the solution blow up is likely to forget it. In particular, he should grasp the three basic concepts of stability, convergence and accuracy. Thereafter the detailed treatment becomes unduly complicated. The average student can discover the relevant methods as and when he needs them; a complete discussion of difference schemes is required only by specialists in computing or in the solution of partial differential equations and the analysis of stability is more suitable for graduates. So it is hard to see how 'science and engineering students in the second and third years of their studies' could have much use for Dr Mitchell's book. (Can they even distinguish between hyperbolic, parabolic and elliptic equations?) Dr Ames more realistically includes among his audience 'graduate students...wishing to know more than the superficial computational aspects of numerical analysis'. (Later his blurb raises false hopes by promising a discussion of Monte Carlo methods.)

Both these books cover parabolic, elliptic and hyperbolic equations, in that order; both give examples of actual computations as well as exercises for the student; neither provides a rigorous treatment of stability and convergence. Mitchell firmly restricts himself to difference schemes; there is a preliminary chapter on matrix methods (which he favours for stability analysis) but no discussion of the classification of partial differential equations or the physical context in which these equations arise. Ames, on the other hand, gives a self-contained account, with an initial chapter on fundamentals and some mention of the physical problems to which the equations correspond. Moreover, he provides a fairly complete set of references (the final chapter alone has three times as many as the whole of Mitchell's book).

The concepts of the truncation error, consistency, convergence and stability are conveniently introduced in connexion with the heat flow equation and the study of implicit methods then leads to Laplace's equation. Both authors inevitably devote a considerable amount of space to elliptic equations, discussing both successive over-relaxation and alternating direction methods (which Mitchell naturally favours). Neither mentions the fast Fourier methods introduced by Hockney for Poisson's equation with simple boundary conditions. For hyperbolic systems they like Lax-Wendroff schemes, though there is no discussion of energy conservation or Arakawa's method. Ames also considers integration along characteristics and Lagrangian difference schemes.

In the pages of discussion on stability, accuracy is largely ignored. With a little ingenuity, instability can be avoided, at least for linear equations. Yet dissipative schemes (Ames mentions them briefly and describes them, regrettably, as overstable) may wipe out the true solution. These errors can be understood as a result of approximating to the wrong differential equation. In a parabolic system, the truncation error reduces the utility of implicit schemes; for hyperbolic equations, accuracy is limited by numerical dispersion at small wavelengths. Thus the number of mesh points limits the range of problems that can be solved—and in two or three space dimensions it is far more expensive to decrease the mesh interval than to increase the number of time steps.

Fluid dynamical computations involve parabolic equations (e.g. Navier–Stokes) which can sometimes be solved as if they were hyperbolic, as well as elliptic equations for the stream function or the pressure. Compressible flow is generally hyperbolic. So the methods in these books are all relevant. In addition, both include special sections on applications to fluid mechanics. Here Ames scores by giving a long list of references, while Mitchell concentrates on a number of specific problems.

Of the two, Ames's book appears fuller and clearer. He relates the equations to physics and also offers clearer explanations of fundamental concepts such as consistency and stability or the Courant–Friedrichs–Lewy criterion. However, the inadequacy of both these books is revealed by comparison with Richtmyer and Morton. The latter not only explain the essential difficulties and describe a variety of difference schemes; they also give a clear mathematical treatment of the relation between convergence and stability, based on functional analysis. Still, they refrain from discussing boundary-value problems and so Ames's book can be recommended for its discussion of elliptic equations and its comprehensive bibliography.

N. O. Weiss